Simplifying Expressions with Roots

EXAMPLE 8.1

Simplify: ⓐ $\sqrt{144}$ ⓑ $-\sqrt{289}$.

$\sqrt{144} = 12$ $\quad$ $-\sqrt{289} = -17$

EXAMPLE 8.2

Simplify: ⓐ $\sqrt{-196}$ ⓑ $-\sqrt{64}$.

$\sqrt{-196}$ = No Real Solution

$= 14i$

$-\sqrt{64} = -8$

We write:	We say:
n^2	*n* squared
n^3	*n* cubed
n^4	*n* to the fourth power
n^5	*n* to the fifth power

Square Roots

$\sqrt{1} = 1$

$\sqrt{4} = 2$

$\sqrt{9} = 3$

$\sqrt{16} = 4$

$\sqrt{25} = 5$

$\sqrt{36} = 6$

$\sqrt{49} = 7$

$\sqrt{64} = 8$

$\sqrt{81} = 9$

$\sqrt{100} = 10$

$\sqrt[2]{121} = 11$

$\sqrt[2]{144} = 12$

$\sqrt[2]{169} = 13$

$\sqrt[2]{196} = 14$

$\sqrt{225} = 15$

$\sqrt{256} = 16$

$\sqrt{289} = 17$

$\sqrt{324} = 18$

$\sqrt{361} = 19$

$\sqrt{400} = 20$

$\sqrt{25} = 5$

$5^2 = \underline{25}$

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$\sqrt[n]{}$

Cube Roots

$(-1)(-1)(-1) = -1$

$\sqrt[3]{1} = 1$

$\sqrt[3]{8} = 2$

$\sqrt[3]{27} = 3$

$\sqrt[3]{64} = 4$

$\sqrt[3]{125} = 5$

$\sqrt[3]{-1} = -1$

$\sqrt[3]{-8} = -2$

$\sqrt[3]{-27} = -3$

$\sqrt[3]{-64} = -4$

$\sqrt[3]{-125} = -5$

Fourth Roots

$\sqrt[4]{1} = 1$

$\sqrt[4]{16} = 2$

$(2)(2)(2)(2) = 16$

$2^4 = 16$

$\sqrt[4]{81} = 3$

$\sqrt[4]{256} = 4$

$\sqrt[4]{625} = 5$

PROPERTIES OF $\sqrt[n]{a}$

When *n* is an even number and

- $a \geq 0$, then $\sqrt[n]{a}$ is a real number.
- $a < 0$, then $\sqrt[n]{a}$ is not a real number.

When *n* is an odd number, $\sqrt[n]{a}$ is a real number for all values of *a*.

EXAMPLE 8.5

Estimate each root between two consecutive whole numbers: ⓐ $\sqrt{105}$ ⓑ $\sqrt[3]{43}$.

[Show/Hide Solution]

a) $\sqrt{105}$ = between 10 + 11

b) $\sqrt[3]{43}$ = between 3 + 4

EXAMPLE 8.7

Simplify: ⓐ $\sqrt{x^2}$ ⓑ $\sqrt[3]{n^3}$ ⓒ $\sqrt[4]{p^4}$ ⓓ $\sqrt[5]{y^5}$.

$\sqrt[2]{x^2} = x$ $\quad \sqrt[3]{n^3} = n$ $\quad \sqrt[4]{p^4} = p$ $\quad \sqrt[5]{y^5} = y$

$\sqrt{x} = x^{\frac{1}{2}}$ $\quad \sqrt[3]{n} = n^{1/3}$ $\quad \sqrt[4]{p} = p^{1/4}$ $\quad \sqrt[5]{y} = y^{1/5}$

$(x^2)^{1/2}$ $\quad (n^3)^{1/3}$

x $\quad n$

EXAMPLE 8.8

Simplify: ⓐ $\sqrt{x^6}$ ⓑ $\sqrt{y^{16}}$.

$\sqrt{x^6}$

$(x^6)^{1/2}$

x^3

$\sqrt{y^{14}}$

$(y^{14})^{1/2}$

y^8

EXAMPLE 8.9

Simplify: ⓐ $\sqrt[3]{y^{18}}$ ⓑ $\sqrt[4]{z^{8}}$.

$\sqrt[3]{y^{18}} = (y^{18})^{1/3} = y^6$

$\sqrt[4]{z^8} = (z^8)^{1/4}$

$= z^2$

EXAMPLE 8.10

Simplify: ⓐ $\sqrt{16n^2}$ ⓑ $-\sqrt{81c^2}$.

$\sqrt{16n^2} = \sqrt{16} \cdot \sqrt{n^2}$

$= 4n$

b) $-\sqrt{81c^2}$

$-\sqrt{81} \cdot \sqrt{c^2}$

$-9c$

EXAMPLE 8.11

Simplify: ⓐ $\sqrt[3]{64p^6}$ ⓑ $\sqrt[4]{16q^{12}}$.

$\sqrt[3]{64p^6} = \sqrt[3]{64} \cdot \sqrt[3]{p^6}$

$= 4 \cdot (p^6)^{1/3}$

$4p^2$

$\sqrt[4]{16q^{12}} = \sqrt[4]{16} \cdot \sqrt[4]{q^{12}}$

$2 \ (q^{12})^{1/4}$

$2q^3$

EXAMPLE 8.12

Simplify: ⓐ $\sqrt{36x^2y^2}$ ⓑ $\sqrt{121a^6b^8}$ ⓒ $\sqrt[3]{64p^{63}q^9}$.

$\sqrt{36x^2y^2} = \sqrt{36} \cdot \sqrt{x^2} \cdot \sqrt{y^2}$

$6xy$

$\sqrt{121a^6b^8} = \sqrt{121} \cdot \sqrt{a^6} \cdot \sqrt{b^8}$

$= 11a^3b^4$

27-49 odd

$\sqrt[3]{64p^{63}q^9} = \sqrt[3]{64} \cdot \sqrt[3]{p^{63}} \cdot \sqrt[3]{q^9}$

$= 4p^{21}q^3$